Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 21338

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

Fifth Semester

Electronics and Communication Engineering

MA 1251 – NUMERICAL METHODS

(Common to Computer Science and Engineering/Information Technology)

(Regulation 2008)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State fixed point theorem.

2. Define diagonally dominant system.

3. State any two properties of divided differences.

4. What are the advantages of Lagrange's formula over Newton?

5. State the trapezoidal rule to evaluate $\int_a^b f(x)\,dx$.

6. State three-point Gaussian quadrature formula.

7. State modified Euler algorithm to solve $y' = f(x, y), y(x_0) = y_0$ at $x_0 = x_0 + h$.

8. State the advantage of Runge-Kutta method over Taylor series method.

9. Write the finite difference equivalent of $\frac{\partial^2 u}{\partial x^2}$ and $\frac{\partial^2 u}{\partial y^2}$.

10. Convert $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ into equivalent finite difference form taking $h = k$ and also write the standard five point formula.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the real positive root of $3x - \cos x - 1 = 0$ by Newton's method correct to 6 decimal places. (8)

(ii) Solve the following set of equations using Gauss-Jordan method

$10x + y + z = 12; 2x + 10y + z = 13; x + y + 5z = 7$. (8)

Or

(b) (i) Find the numerically largest eigenvalue of $\begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$ by using power method. (8)

(ii) Solve the following set of equations using Gauss Seidal iterative procedure

$20x + y - 2z = 17; 3x + 20y - z = -18; 2x - 3y + 20z = 25$. (8)

12. (a) (i) Find $f(8)$ by Newton's divided difference formulae for the data : (8)

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

(ii) Find the polynomial $f(x)$ by using Lagrange's formula and hence find $f(3)$ for (8)

x :	0	1	2	5
$y = f(x)$:	2	3	12	147

Or

(b) (i) Using Newton's forward interpolation formula, find the polynomial $f(x)$ satisfying the following data. Hence find $f(5)$. (8)

x :	4	6	8	10
$f(x)$:	1	3	8	10

(ii) Obtain the cubic spline approximation for the function $y = f(x)$ from the following data, given that $y_0'' = y''_3 = 0$. (8)

x :	–1	0	1	2
$y = f(x)$:	–1	1	3	35

13. (a) (i) For the given data

x :	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$y = f(x)$:	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.1$. (8)

(ii) Evaluate the integral $\int_1^2\int_1^2 \frac{dxdy}{(x^2+y^2)}$ by taking $h = 0.2$ alone x–direction and $k = 0.25$ along y–direction and by using trapezoidal rule. (8)

Or

(b) (i) Evaluate $\int_0^1 \frac{dx}{\sqrt{1+x^4}}$ by using the three point Gaussian quadrature formula. (8)

(ii) Evaluate $\int_0^2 \frac{dx}{x^2+4}$ using Romberg's method. Hence obtain an approximate value for π. (8)

14. (a) (i) Evaluate the values of $y(0.1)$ and $y(0.2)$ given $y''-(xy')^2 + y^2 = 0; y(0) = 1$, $y'(0) = 0$ by using Taylor series method. (8)

(ii) Using Milne's method, find $y(0.8)$ if $y(x)$ is the solution of $\frac{dy}{dx} = x^3 + y$ given $y(0) = 2$, $y(0.2) = 2.073$, $y(0.4) = 2.452$ $y(0.6) = 3.023$ taking $h = 0.2$. (8)

Or

(b) (i) Using Runge-Kutta method of order four solve $\frac{dy}{dx} = x + y^2$ with $y(0) = 1$ at $x = 0.1$, $x = 0.2$ with $h = 0.1$. (8)

(ii) Using Adams-Bashforth method find $y(4.4)$ given $5x\frac{dy}{dx} + y^2 = 2$ given that $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$, $y(4.3) = 1.0143$. (8)

15. (a) (i) Using the finite difference method, solve $\frac{d^2y}{dx^2} - y = 2$ subject to the boundary conditions $y(0) = 0$ and $y(1) = 1$, by taking $h = 0.25$. (8)

(ii) Solve $\frac{\partial^2 u}{\partial x^2} - 32\frac{\partial u}{\partial t} = 0$, $t > 0$, $0 < x < 1$ given that $u(0,t) = 0, u(1,t) = t$, $u(x,0) = 0$. Assume $h = 0.25$ and find the values of u upto $t = 5$. (8)

Or

(b) (i) Solve $y_{u} = y_{xx}$ up to $t = 0.5$ with a spacing of 0.1 subject to $y(0,t) = y(1,t)$, $y_t = (x,0) = 0$ and $y(x,0) = 10 + x(1 + x)$. (8)

(ii) Solve the Poisson equation $u_{xx} + u_{yy} + 10(x^2 + y^2 + 10)$ over the square with sides $x = 0$, $y = 0$, $x = 3$, $y = 3$ with $u = 0$ on the boundary, taking $h = 1$. (8)
